

QCD symmetries and the η and η' in nuclei

Steven D. Bass

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Abstract We discuss the role of QCD symmetries in understanding the η and η' mesons in nuclear media. Recent results on the η' mass in nuclei from the CBELSA/TAPS collaboration are very similar to the prediction of the Quark Meson Coupling model.

Keywords Eta prime meson · Medium modifications · Mesic nuclei

1 Introduction

Recent progress in theoretical and experimental studies of the η - and η' - (as well as pion and kaon) nucleus systems promises to yield valuable new information about dynamical chiral and axial U(1) symmetry breaking in low energy QCD [1]. With increasing nuclear density chiral symmetry is partially restored corresponding to a reduction in the values of the quark condensate and pion decay constant f_π [2, 3]. This in turn leads to changes in the properties of hadrons in medium including the masses of the Goldstone bosons. While pions and kaons are would-be Goldstone bosons associated with chiral symmetry, the isosinglet η and η' mesons are too massive by about 300-400 MeV for them to be pure Goldstone states. They receive extra mass from non-perturbative gluon dynamics associated with the QCD axial anomaly; for recent reviews see [1, 4]. How does this gluonic part change in nuclei? Medium modifications need to be understood self-consistently within the interplay of confinement, spontaneous chiral symmetry breaking and axial U(1) dynamics.

The η - and η' -nucleon interactions are believed to be attractive corresponding to a reduced effective mass in the nuclear medium and the possibility that these mesons might form strong-interaction bound-states in nuclei. For the η one finds a sharp rise in the cross section at threshold for η production in both

S. D. Bass
Stefan Meyer Institute for Subatomic Physics, Austrian Academy of Sciences,
Boltzmanngasse 3, A 1090 Vienna, Austria

photoproduction from ^3He [5] and in proton-deuteron collisions [6] which may hint at a reduced η effective mass in the nuclear medium. Measurement of the η' -nucleus optical potential by the CBELSA/TAPS collaboration suggests that the effective η' mass drops by about 40 MeV at nuclear matter density [7]. For the pion and kaon systems one finds a small pion mass shift of order a few MeV in nuclear matter [2] whereas kaons are observed to experience an effective mass drop for the K^- to about 270 MeV at two times nuclear matter density in heavy-ion collisions [8,9]. The same heavy-ion experiments also suggest the effective mass of anti-protons is reduced by about 100-150 MeV below their mass in free space [8]. The η -nucleon interaction is characterised by a strong coupling to the $S_{11}(1535)$ nucleon resonance. For example, η meson production in proton nucleon collisions close to threshold is known to proceed via a strong isovector exchange contribution with excitation of the $S_{11}(1535)$ [10]. Recent measurements of η' production suggest a different mechanism for this meson [11]. Experiments in heavy-ion collisions [12] and η photoproduction from nuclei [13,14] suggest little modification of the $S_{11}(1535)$ excitation in-medium, though some evidence for the broadening of the S_{11} in nuclei was reported in [14].

There is presently vigorous experimental [7,15,16,17,18,19] and theoretical [1,20,21,22,23] activity aimed at understanding the η and η' in medium and to search for evidence of possible η and η' bound states in nuclei. QCD inspired models of the η and η' nucleus systems are constructed with different selections of “good physics input”: how they treat confinement, chiral symmetry and axial U(1) dynamics. Here we focus on the Quark Meson Coupling model (QMC, for a review see [24]). In the QMC model medium modifications are calculated at the quark level through coupling of the light quarks in the hadron to the scalar isoscalar σ (and also ω and ρ) mean fields in the nucleus. Possible binding energies and the in-medium masses of the η and η' are sensitive to the flavour-singlet component in the mesons and hence to the non-perturbative glue associated with axial U(1) dynamics [20].

Meson masses in nuclei are determined from the meson nucleus optical potential and the scalar induced contribution to the meson propagator evaluated at zero three-momentum, $\mathbf{k} = 0$, in the nuclear medium. Let $k = (E, \mathbf{k})$ and m denote the four-momentum and mass of the meson in free space. Then, one solves the equation

$$k^2 - m^2 = \text{Re } \Pi(E, \mathbf{k}, \rho) \quad (1)$$

for $\mathbf{k} = 0$ where Π is the in-medium s -wave meson self-energy. Contributions to the in medium mass come from coupling to the scalar σ field in the nucleus in mean-field approximation, nucleon-hole and resonance-hole excitations in the medium. For $\mathbf{k} = 0$, $k^2 - m^2 \sim 2m(m^* - m)$ where m^* is the effective mass in the medium. The mass shift $m^* - m$ is the depth or real part of the meson nucleus optical potential. The imaginary part of the potential measures the width of the meson in the nuclear medium. The s -wave self-energy can be

written as [25]

$$\Pi(E, \mathbf{k}, \rho) \Big|_{\{\mathbf{k}=0\}} = -4\pi\rho \left(\frac{b}{1 + b\langle\frac{1}{r}\rangle} \right). \quad (2)$$

Here ρ is the nuclear density, $b = a(1 + \frac{m}{M})$ where a is the meson-nucleon scattering length, M is the nucleon mass and $\langle\frac{1}{r}\rangle$ is the inverse correlation length, $\langle\frac{1}{r}\rangle \simeq m_\pi$ for nuclear matter density. Attraction corresponds to positive values of a . The denominator in Eq.(2) is the Ericson-Ericson-Lorentz-Lorenz double scattering correction.

Meson mass shifts in medium can be investigated through studies of excitation functions in photoproduction experiments from nuclear targets and through searches for possible meson bound states in nuclei. In photoproduction experiments the production cross section is enhanced with the lower effective meson mass in the nuclear medium. When the meson leaves the nucleus it returns on-shell to its free mass with the energy budget conserved at the expense of the kinetic energy so that excitation functions and momentum distributions can provide essential clues to the meson properties in medium [26]. Using this physics a first (indirect) estimate of the η' mass shift has recently been deduced by the CBELSA/TAPS Collaboration [7]. The η' -nucleus optical potential $V_{\text{opt}} = V_{\text{real}} + iW$ deduced from these photoproduction experiments is

$$\begin{aligned} V_{\text{real}}(\rho_0) &= m^* - m = -37 \pm 10(\text{stat.}) \pm 10(\text{syst.}) \text{ MeV} \\ W(\rho_0) &= -10 \pm 2.5 \text{ MeV} \end{aligned} \quad (3)$$

at nuclear matter density ρ_0 . In this experiment the average momentum of the produced η' was 1.1 GeV and the mass shift was measured in production from a carbon target. The mass shift, Eq.(3), is very similar to the expectations of the Quark Meson Coupling model, see below. If substituted into Eq.(2) with the Ericson-Ericson denominator switched off, then one finds an effective scattering length with real part of 0.5 fm. The COSY-11 collaboration have recently determined the η' -nucleon scattering length in free space to be

$$\begin{aligned} \text{Re}(a_{\eta'p}) &= 0 \pm 0.43 \text{ fm} \\ \text{Im}(a_{\eta'p}) &= 0.37^{+0.40}_{-0.16} \text{ fm} \end{aligned} \quad (4)$$

from studies of the final state interaction in η' production in proton-proton collisions close to threshold [27]. Theoretical models in general prefer a positive sign for the real part of $a_{\eta'p}$.

New experiments are planned to look for possible η' bound states in carbon using the (p, d) reaction at GSI [17] and in photoproduction at ELSA [18]. The small η' width in nuclei 20 ± 5.0 MeV at nuclear matter density in Eq.(3) was extracted from measurements of the transparency ratio for η' photoproduction from nuclear targets [15] and suggests the possibility of relatively narrow bound η' -nucleus states accessible to experiments. For clean observation of a bound state one needs the real part of the optical potential to be much bigger than the imaginary part. COSY searches are focussed on possible η bound states in ^3He and ^4He [16].

2 QCD symmetries and the η and η'

Spontaneous chiral symmetry breaking in QCD induces an octet of Goldstone bosons associated with SU(3) and also (before extra gluonic effects in the singlet channel) a flavour-singlet Goldstone boson. The mass squared of these Goldstone bosons is proportional to the current mass of their valence quarks. While the pion and kaon fit well in this picture, to understand the isosinglet η and η' masses one needs extra mass in the flavour-singlet channel associated with non-perturbative topological gluon configurations [4, 28], related perhaps to confinement [29] or instantons [30]. The gluonic mass term $\tilde{m}_{\eta_0}^2$ satisfies the Witten-Veneziano mass formula [31, 32]

$$m_\eta^2 + m_{\eta'}^2 = 2m_K^2 + \tilde{m}_{\eta_0}^2 \quad (5)$$

and has a rigorous interpretation in terms of the QCD Yang-Mills topological susceptibility. SU(3) breaking generates mixing between the octet and singlet states which, together with the gluonic mass contribution, yields the massive η and η' bosons. Phenomenological studies of various decay processes give a value for the η - η' mixing angle between -15° and -20° [33]. In the OZI limit of no gluonic mass term the η would be approximately an isosinglet light-quark state ($\frac{1}{\sqrt{2}}|\bar{u}u + \bar{d}d\rangle$) with mass $m_\eta \sim m_\pi$ degenerate with the pion and the η' would be a strange-quark state $|\bar{s}s\rangle$ with mass $m_{\eta'} \sim \sqrt{2m_K^2 - m_\pi^2}$, mirroring the isoscalar vector ω and ϕ mesons.

The gluonic mass term is related to the QCD axial anomaly in the divergence of the flavour-singlet axial-vector current. While the non-singlet axial-vector currents are partially conserved (they have just mass terms in the divergence), the singlet current $J_{\mu 5} = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s$ satisfies the anomalous divergence equation

$$\partial^\mu J_{\mu 5} = 6Q + \sum_{k=1}^3 2im_k \bar{q}_k \gamma_5 q_k \quad (6)$$

where $Q = \partial^\mu K_\mu = \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}$ is the topological charge density. The integral over space $\int d^4z Q = n$ measures the gluonic winding number [28] which is an integer for (anti-)instantons and which vanishes in perturbative QCD.

η - η' mixing means that non-perturbative glue through axial U(1) dynamics plays an important role in both the η and η' mesons and their interactions. The anomalous glue that generates the large η and η' masses also drives OZI violating η and η' production and decay processes [33-37] and enters in the η' -nucleon interaction [38] that has been the subject of vigorous experimental investigation at COSY [39]. The QCD axial anomaly also plays an important role in interpretation of the nucleon's flavour-singlet axial-charge (or "quark spin content") measured in polarised deep inelastic scattering and associated with the proton spin puzzle [40, 41].

Within the low energy effective chiral Lagrangian for QCD the gluonic mass term $\tilde{m}_{\eta_0}^2$ is introduced via a flavour-singlet potential involving the topological

charge density Q which is constructed so that the Lagrangian also reproduces the axial anomaly [34]. In this approach the medium dependence of $\tilde{m}_{\eta_0}^2$ is introduced through coupling to the σ (correlated two-pion) mean-field in the nucleus through the interaction term $\mathcal{L}_{\sigma Q} = g_{\sigma Q} Q^2 \sigma$ where $g_{\sigma Q}$ denotes coupling to the σ mean field. One finds the gluonic mass term decreases in-medium $\tilde{m}_{\eta_0}^{*2} < \tilde{m}_{\eta_0}^2$ independent of the sign of $g_{\sigma Q}$. The medium acts to partially neutralise axial U(1) symmetry breaking by gluonic effects [20].

As a second interesting application of the QCD effective Lagrangian approach, the OZI violating interaction $\lambda Q^2 \partial_\mu \pi_a \partial^\mu \pi_a$ with π_a the pseudoscalar Goldstone fields is needed to generate the leading (tree-level) contribution to the decay $\eta' \rightarrow \eta \pi \pi$ [35]. When iterated in the Bethe-Salpeter equation for $\eta' \pi$ rescattering this interaction yields a dynamically generated resonance with quantum numbers $J^{PC} = 1^{-+}$ and mass about 1400 MeV. The generation of this state is mediated by the OZI violating coupling of the η' [37]. One finds a possible dynamical interpretation of the light-mass 1^{-+} exotics observed in experiments at BNL [42] and CERN [43]. This OZI violating interaction will also contribute to higher L odd partial waves with quantum numbers L^{-+} . These states are particularly interesting because the quantum numbers $1^{-+}, 3^{-+}, 5^{-+} \dots$ are inconsistent with a simple quark-antiquark bound state. The COMPASS experiment at CERN has recently measured exclusive production of $\eta' \pi^-$ and $\eta \pi^-$ in 191 GeV π^- collisions on a hydrogen target [44]. They find the interesting result that $\eta' \pi^-$ production is enhanced relative to $\eta \pi^-$ production by a factor of 5-10 in the exotic $L = 1, 3, 5$ partial waves with quantum numbers L^{-+} in the inspected invariant mass range up to 3 GeV. No enhancement was observed in the even L partial waves.

3 The η and η' in nuclei

The physics of the η and η' in medium has been investigated by Bass and Thomas [20] within the Quark Meson Coupling model [24, 45, 46] taking into account η - η' mixing and the flavour-singlet component in these mesons. In these calculations the large η and η' masses are used to motivate taking an MIT Bag description for the meson wavefunctions. Gluonic topological effects are understood to be “frozen in”, meaning that they are only present implicitly through the masses and mixing angle in the model. The in-medium mass modification comes from coupling the light (up and down) quarks and antiquarks in the meson wavefunction to the scalar σ mean-field in the nucleus working in mean-field approximation [24]. The coupling constants in the model for the coupling of light-quarks to the σ (and ω and ρ) mean-fields in the nucleus are adjusted to fit the saturation energy and density of symmetric nuclear matter and the bulk symmetry energy. The strange-quark component of the wavefunction does not couple to the σ field and η - η' mixing is readily built into the model. Gluon fluctuation and centre-of-mass effects are assumed to be independent of density. The model results for the meson masses in medium and the real part of the meson-nucleon scattering lengths are shown in Table

Table 1 Physical masses fitted in free space, the bag masses in medium at normal nuclear-matter density, $\rho_0 = 0.15 \text{ fm}^{-3}$, and corresponding effective meson-nucleon scattering lengths. The values of $\text{Re}a_\eta$ are obtained with the Ericson-Ericson denominator turned-off (since we work in mean-field approximation).

	m (MeV)	m^* (MeV)	$\text{Re}a$ (fm)
η_8	547.75	500.0	0.43
η (-10°)	547.75	474.7	0.64
η (-20°)	547.75	449.3	0.85
η_0	958	878.6	0.99
η' (-10°)	958	899.2	0.74
η' (-20°)	958	921.3	0.47

1 for different values of the η - η' mixing angle which is taken to be density independent.

With an η - η' mixing angle of -20° the QMC prediction for the η' mass in medium at nuclear matter density is 921 MeV, that is a mass shift of -37 MeV. This value is in excellent agreement with the mass shift $-37 \pm 10 \pm 10$ MeV deduced from photoproduction data [7]. Mixing increases the octet relative to singlet component in the η' , reducing the binding through increased strange quark component in the η' wavefunction. Without the gluonic mass contribution the η' would be a strange quark state after η - η' mixing. Within the QMC model there would be no coupling to the σ mean field and no mass shift so that any observed mass shift is induced by glue associated with the QCD axial anomaly that generates part of the η' mass.

Increasing the flavour-singlet component in the η at the expense of the octet component gives more attraction, more binding and a larger value of the η -nucleon scattering length, $a_{\eta N}$. η - η' mixing with the phenomenological mixing angle -20° leads to a factor of two increase in the mass-shift and in the scattering length obtained in the model relative to the prediction for a pure octet η_8 . This result may explain why values of $a_{\eta N}$ extracted from phenomenological fits to experimental data where the η - η' mixing angle is unconstrained [47] give larger values (with real part about 0.9 fm) than those predicted in theoretical coupled channels models where the η is treated as a pure octet state [48, 49].

For baryons in symmetric nuclear matter the QMC model predicts an effective proton mass about 755 MeV at nuclear matter density [24]. The S_{11} is interpreted in quark models as a 3-quark state $(1s)^2(1p)$. In QMC one finds an excitation energy of ~ 1544 MeV, consistent with observations, with the scalar attraction compensated by repulsion from coupling to the ω mean-field to give the excitation energy [20]. Small mass shift is also found in coupled channels models where the S_{11} is instead interpreted as a $K\Sigma$ quasi-bound state, with the η instead treated as a pure octet state [50].

For the η' in medium, larger mass shifts, downwards by up to 80-150 MeV, were found in recent Nambu-Jona-Lasinio model calculations (without confinement) [21] and in linear sigma model calculations (in a hadronic basis)

[22] which also suggest a rising η effective mass at finite density. A chiral coupled channels calculation performed with possible η' -nucleon scattering lengths with real part between 0 and 1.5 fm is reported in [51].

4 Outlook

Medium modifications of hadron properties are determined by chiral and flavour symmetries in QCD. The η and η' are sensitive to flavour-singlet axial U(1) degrees of freedom. QCD inspired models including confinement, chiral and axial U(1) dynamics yield a range of predictions for the η and η' mass shifts in nuclei and the corresponding meson-nucleon scattering lengths. The QMC prediction for the η' mass shift is very similar to the recent value determined by CBELSA/TAPS from photoproduction experiments. The model value for the real part of the η -nucleon scattering length is also close to values extracted from phenomenological fits to low-energy scattering data. New data on the η and η' in nuclei and possible bound states are expected soon from running and planned experiments at COSY, ELSA and GSI, and will help further constrain our understanding of axial U(1) dynamics in low-energy QCD.

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